B.Sc. Physics (CBCS) Semester 1
Core course 1
paper II
Full Marks: 40

Question 1: Answer any five (5) questions: **[5 X 1 = 5]**

i) What is an inertial frame of reference?

ii) Write down Galilean transformation equations for space and time.

iii) What is the value of coefficient of restitution "e" for a perfectly elastic collision?

iv) Sate the relation between elastic constants Y, η and σ, where the symbols have their usual meanings.

v) What is the potential energy of an object under elastic deformation?

vi) Which of the following projectiles will have maximum range: angles are 30°, 45°, 70°, and 90°?

vii) What is the kinetic energy of a cylinder rolling down an inclined plane?

viii) What is the period of geostationary satellite?

Question 2: Answer any three (3) questions: **[3 X 5 = 15]**

i) State Perpendicular Axis Theorem. Calculate the moment of inertial of a circular disc about one of its diameter. [2+3]

ii) State Hooke's law in elasticity. Show that for an elastic body undergoing longitudinal strain, the elastic potential energy per unit volume = $\frac{1}{2} \times stress \times strain$ [2+3]

iii) Define sharpness of resonance. What are half power frequencies? [3+2]

iv) Obtain the expression for the Coriolis force experienced in a coordinate frame rotating with constant angular velocity about z – axis. [5]

v) Derive Lorentz transformation equations for space and time. [5]

Question 3: Answer any two (2) questions: **[2 X 10 = 20]**

i) Define viscosity of a fluid. Derive the Poiseuille's equation for the flow of viscous fluid through a capillary stating clearly the assumptions involved. [2+ 8]

ii) Show how a two body problem involving central force can be reduced to a one body problem. What would be the expression for angular momentum and energy? [5+5]

iii) State the postulates of Einstein's theory of relativity. Explain the physical significance of the null result of Michelson – Morley experiment. Show that if (x_1, y_1,

z_1, t_1) and (x_2, y_2, z_2, t_2) are the co- ordinates of one event in S_1 and the corresponding event in S_2 respectively, then the expression –

$ds_1^2 = dx_1^2 + dy_1^2 + dz_1^2 - c^2 dt_1^2$

is an invariant in Lorentz transformation of co-ordinates.

Show that for a relativistic particle, the product of particle velocity and the phase velocity of the associated de Broglie wave is c^2? [2+3+3+2]

iv) Show that the expression for the force observed in rotating co-ordinate system is :

$$\vec{F}_{rot} = m\vec{a}_{in} - m\,[2\,\vec{\omega} \times \vec{v}_{rot} + \vec{\omega} \times (\vec{\omega} \times \vec{r})]$$

Explain the different terms on the right hand side of the equation.

Answers:

Q1 i) the reference frames non-accelerated with respect to one another and in which Newton's second law remains valid, is called an inertial reference frame.

ii) Let S'-frame moves with uniform velocity 'v' along x-axis and S-frame is at rest. Let the origin of the two frames coincide at time t = 0. Then the transformation equations from S'(x', y', z', t') to S(x, y, z, t) are written as:

$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

iii) The value of coefficient of restitution 'e' for perfectly elastic collision is 1.

iv) The relation between Y, η and σ is $Y = 2\eta\,(1+\sigma)$.

v) The potential energy of an object under elastic deformation is $\frac{1}{2} \times stress\ \times strain.$

Q2 i) Let us consider a ring between radii r and $r+dr$ and of thickness t. The mass of the ring is

$dm = 2\,\pi\, r\, dr\, t\, \rho$ where ρ = density of the ring.

Therefore, the moment of inertia of the circular disc about z-axis is

$$\begin{aligned} I_z &= \int_0^a r^2\, dm \\ &= \int_0^a 2\pi r^3 \rho t\, dr \\ &= 2\pi\rho t \frac{a^4}{4} \\ &= (\pi a^2 t\rho)\frac{a^2}{2} \\ &= \frac{M\,a^2}{2} \end{aligned}$$

Perpendicular axis theorem: This theorem enables us to determine the moment of inertia of a laminar body about an axis perpendicular to the plane of the lamina (I_z) which is equal to the sum of the moments of inertia about two axes mutually at right angles to each other and lying on the plane of the lamina [I_x and I_y].

Mathematically, $I_z = I_x + I_y$.

Q2 ii) Hooke's law: For any elastic deformation, the stress is proportional to the strain, within a certain limit of stress, called the elastic limit. i.e.

$$\frac{stress}{strain} = constant$$

Let a wire of length L be stretched be an amount l, by a stretching force F. Let the extension l occurred in infinitesimally small steps of dl, due to the applied force, within the elastic limit. Therefore, work done in the process is given by,

$W = \int_0^l F\, dl = \frac{Y\alpha}{L}\int_0^l l\, dl = \frac{Y\alpha l^2}{2L}$ -----(1)

Where Y = Young's modulus = $\frac{F/\alpha}{l/L}$, α being the cross section of the wire.

Therefore, from equation (1) we can write

$$W = \frac{1}{2} \times Y \times \left(\frac{l}{L}\right)^2 \times (\alpha L)$$

Thus, the strain energy per unit volume is $= \frac{W}{\alpha L}$

$$= \frac{1}{2} \times Y \times \left(\frac{l}{L}\right)^2$$

$$= \frac{1}{2} \times \left(Y \times \frac{l}{L}\right) \times \frac{l}{L}$$

$$= \frac{1}{2} \times stress \times strain$$ (proved)

Q2 iii) Average power of the driver in forced vibration over a complete cycle is given by

$P_{av} = \frac{P^2\, k}{2[k^2 + (\omega m - s/\omega)^2]}$ ---- (1)

Where ω = angular frequency of the driver.

k = mechanical resistance

s = stiffness factor

m = mass of the particle

F = amplitude of the driving force

At resonance, $\omega_0 m = s/\omega_0$

or, $\omega_0 = \sqrt{s/m} = resonant\ frequency$

and P_{av} maximum = $F^2/2k$

$(P_{av})_{resonance} = F^2/2k$ -----(2)

When k is small, the resonance peak of P_{av} is high, and vice versa. Thus resonance is sharp when damping is small. For large damping, the resonance is broad or flat.

Thus, the sharpness of resonance gives the rapidity with which the average power P_{av} supplied by the driver drops off as ω differs from its value at resonance.

Therefore, $\frac{P_{av}}{(P_{av})_r} = \frac{k^2}{k^2 + m^2\left(\omega - \frac{\omega_0^2}{\omega}\right)^2}$

$$= \frac{4b^2}{4b^2 + \omega_0^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}$$

$$= \frac{4b^2}{4b^2 + \omega_0^2\Delta^2}$$

Where, $\Delta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$

At, $\omega = \omega_0$, $\Delta = 0$ and $\frac{P_{av}}{(P_{av})_r} = 1$

The fig shows that for a given value of k and b, there are two values of ω, namely ω_1 and ω_2 for which the average power P_{av} is half of its value at resonance. These frequencies are called the half power frequencies.

$\omega_1 < \omega_0$; lower half power frequency,
$\omega_2 > \omega_0$; upper half power frequency.

Q3. i) Viscosity: When there is a relative motion between successive layers of a fluid, the faster moving layer is acted on by a retarding force while the slower moving one gets accelerated leading towards uniform velocity gradient. This property of fluid to oppose relative movements of its parts, i.e. internal friction, is called viscosity.

Poiseuille's equation for the flow of viscous fluid through a capillary:

Statement: The equation states that the flux of the liquid is proportional to the pressure gradient and to the fourth power of the radius of the tube.

Poiseuille's equation determines the flow of a liquid through a narrow tube. The deduction is based on the **following assumptions**:

a) The liquid in contact with the walls of the tube is at rest (no slip condition assumed).
b) The motion is streamline parallel to the axis of the tube, and hence, the pressure will be constant over any given cross-section.
c) The pressure difference at the ends has been fully used up in overcoming viscosity of the fluid.

Derivation:

Let AB represent a narrow tube and XX', the axis of the tube. Let the velocity at a distance 'r' from the axis of the tube be 'u', when a steady state is reached. Since the liquid adheres to the walls and the velocity in contact with the walls drops to zero, the velocity decreases as distance from the axis increases and hence, velocity gradient du/dr is negative.

Let 'l' be the length of the tube AB of internal radius 'a'; p_1, p_2 be pressures at the two ands. The pressure is a function of 'x' alone and the pressure gradient $\frac{p_1 - p_2}{l}$ is constant. If F_1 and F_2 be the frictional forces on the liquid between two cylinders of length dx and radii r and $r+dr$, then,

$$F_1 = -\eta \mathrm{A} \frac{\mathrm{du}}{\mathrm{dr}} = -\eta \; 2\pi \mathrm{rdx} \; \frac{\mathrm{du}}{\mathrm{dr}}$$

$$F_2 = \eta \; 2\pi \mathrm{rdx} \; \frac{\mathrm{du}}{\mathrm{dr}} + \eta \; \frac{\mathrm{d}}{\mathrm{dr}}\left[(2\pi \mathrm{rdx}) \; \frac{\mathrm{du}}{\mathrm{dr}}\right] \delta \mathrm{r}$$

The total force on the liquid between radii r and $r+dr$ is,

$F = F_1 + F_2$

$$= \eta \frac{\mathrm{d}}{\mathrm{dr}} \left[(2\pi \mathrm{rdx}) \frac{\mathrm{du}}{\mathrm{dr}}\right] \delta \mathrm{r}$$

$$= 2\pi\eta \, \mathrm{dx} \frac{\mathrm{d}}{\mathrm{dr}} \left(\mathrm{r} \frac{\mathrm{du}}{\mathrm{dr}}\right) \delta \mathrm{r}$$

In steady condition, these forces of viscosity must be balanced by pressure force which is the pressure difference on the plane ends of the liquid mass.

Therefore, $2\pi\eta \, dx \frac{d}{dr}\left(r \frac{du}{dr}\right) \delta r = \frac{dp}{dx} \, dx \, 2\pi r \delta r$

or, $\eta \frac{d}{dr}\left(r \frac{du}{dr}\right) = r \frac{dp}{dx}$

or, $\eta \frac{d}{dr}\left(r \frac{du}{dr}\right) \int_0^l dx = r \int_{p_1}^{p_2} dp$

or, $\eta \frac{d}{dr}\left(r \frac{du}{dr}\right) l = r(p_1 - p_2)$

Therefore, $\frac{d}{dr}\left(r \frac{du}{dr}\right) = -\frac{p_1 - p_2}{\eta l} r$

Integrating the above equation with respect to *r*,

$$r \frac{du}{dr} = -\frac{(p_1 - p_2)}{\eta l} \frac{r^2}{2} + C_1$$

Or, $\frac{du}{dr} = -\frac{(p_1 - p_2)}{\eta l} \frac{r}{2} + \frac{C_1}{r}$

Integrating again, $u = -\frac{(p_1 - p_2)}{\eta l} \frac{r^2}{4} + C_1 \log r + C_2$

Now, $u \neq \infty$ when $r = 0$, $C_1 = 0$

Again, u = 0 when r = a

Therefore $0 = \frac{-(p_1 - p_2)}{\eta l} \frac{a^2}{4} + C_2$ thus $C_2 = \frac{(p_1 - p_2)}{\eta l} \frac{a^2}{4}$

So, $u = \frac{p_1 - p_2}{4\eta l} (a^2 - r^2)$

$= A(a^2 - r^2)$ where $A = \frac{p_1 - p_2}{4\eta l}$

Thus, we see that at $r = 0$, the flow velocity is maximum, $u_{max} = \frac{p_1 - p_2}{4\eta l} a^2$ along the axis of the tube ($r = 0$)

The liquid current through the tube is,

$$I = \iint \rho\vec{u} \cdot \overrightarrow{ds}$$

$$= \iint \rho u \, ds$$

$$= 2\pi\rho \int_0^a ru \, dr$$

$$= 2\pi\rho \int_0^a \frac{p_1 - p_2}{4\eta l} r\,(a^2 - r^2)\, dr$$

$$= 2\pi\rho \frac{p_1 - p_2}{4\eta l} \frac{a^4}{4}$$

Volume flowing out per sec,

$$V = \frac{I}{P} = \frac{2\pi\,(p_1 - p_2)\,a^4}{4\eta l \quad 4} = \frac{\pi\,(p_1 - p_2)\,a^4}{8\eta l}$$

This is Poiseuille's equation.

Q3 iii) Postulates of Einstein's theory of relativity are as follows:

a) The laws of physics are invariant in all inertial systems.
b) The speed of light in vacuum is the same for all observers, regardless of the motion of the light sources.

The physical significance of null result of Michelson-Morley experiment is as follows:

The null result of Michelson- Morley experiment establishes that

i. Ether does not exist as no deflection in light due to ether was detected.
ii. Speed of Light is constant irrespective of relative motion of light emitting body and observer.

Invariance of ds^2 : −

The Lorentz transformation equations from S' (x_2, y_2, z_2, t_2) frame (moving with uniform velocity 'v' along x-axis) to S-frame (x_1, y_1, z_1, t_1) (at rest) are:

$$x_2 = \frac{x_1 - vt_1}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}; \quad y_2 = y_1; \quad z_2 = z_1; \quad t_2 = \frac{t_1 - {x_1 v}/{c^2}}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

Therefore:

$$dx_2 = \frac{dx_1 - vdt_1}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}; \qquad dy_2 = dy_1; \qquad dz_2 = dz_1; \qquad dt_2 = \frac{dt_1 - {}^{vdx_1}/_{c^2}}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

So,

$ds_2{}^2 = dx_2{}^2 + dy_2{}^2 + dz_2{}^2 - c^2 dt_2{}^2$

$= \left(\frac{dx_1 - vdt_1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}\right)^2 + dy_1{}^2 + dz_1{}^2 - c^2\left(\frac{dt_1 - {}^{vdx_1}/_{c^2}}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}\right)^2$

$= \frac{1}{1-\left(\frac{v^2}{c^2}\right)}\left[dx_1{}^2 - 2dx_1dt_1v + v^2dt_1{}^2 - c^2dt_1{}^2 - 2\,{}^{v}/_{c^2}\,c^2dx_1dt_1 + c^2\frac{v^2}{c^4}\,dx_1{}^2\right] + dy_1{}^2 + dz_1{}^2$

$= \frac{1}{1-\left(\frac{v^2}{c^2}\right)}\left[\left\{1 - \left(\frac{v^2}{c^2}\right)\right\}dx_1{}^2 - c^2\left\{1 - \left(\frac{v^2}{c^2}\right)\right\}dt_1{}^2\right] + dy_1{}^2 + dz_1{}^2$

$= = dx_1{}^2 + dy_1{}^2 + dz_1{}^2 - c^2dt_1{}^2.$ (Proved)

To show that for a relativistic particle, the product of particle velocity and the phase velocity of the associated de Broglie wave is *c*^{2}:

If η = frequency and λ = wavelength of the particle,

Particle velocity = $\eta\lambda$

Let phase velocity of the particle be v.

Therefore, particle velocity × phase velocity = $\eta\lambda \times v$

$= \eta\lambda \times {}^{h}/_{m\lambda}$ [Since $\lambda = \frac{h}{mv}$]

$= {}^{\eta h}/_{m}$

$= {}^{E}/_{m} = c^2$ (Proved).